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First Semester B.E. Degree Examination, June/July 2023 Engineering Mathematics - I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find n^{th} derivation of $\frac{x^3}{x^2-1}$. (05 Marks)
- b. Find Pedal equation for $\frac{2a}{r} = 1 - \cos \theta$. (05 Marks)
- c. Find radius of curvature for $y^2 = \frac{a^2(a-x)}{x}$, where the curve cuts x - axis. (06 Marks)
- OR**
- 2 a. If $y = \sin(m \cos^{-1} x)$, Show that $(1-x^2) Y_{n+2} - (2n+1)x Y_{n+1} + (n^2 - m^2) Y_n = 0$. (06 Marks)
- b. ST curves $r^n = a^n \sec \theta$, $r^n \sin \theta = b^n$ cuts orthogonally. (05 Marks)
- c. If $\sqrt{r} \cos\left(\frac{\theta}{2}\right) = a$, Show that $\rho = \frac{2}{a} r^{3/2}$. (05 Marks)

Module-2

- 3 a. If $u = f\left(\frac{y-x}{xy}, \frac{x-z}{xz}\right)$, show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$. (06 Marks)
- b. Evaluate $\lim_{x \rightarrow 0} \left[\frac{\tan x}{x} \right]^{1/x^2}$. (05 Marks)
- c. Obtain the expansion of $\tan x$ in terms of powers of $\left(x - \frac{\pi}{4}\right)$ upto 3rd degree. (05 Marks)

OR

- 4 a. For $y = \log_e(1+x)$, find Maclaurin's series expansion upto 4th degree, hence evaluate $\log_e 2$. (06 Marks)
- b. If $u = \sin^{-1} \left[\frac{x^2 + y^2}{\sqrt{x} - \sqrt{y}} \right]$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{3}{2}$ tones. (05 Marks)
- c. If $x + y + z = u$, $y + z = v$, $z = uvw$.
 $ST J \left[\frac{x, y, z}{u, v, w} \right] = uv$. (05 Marks)

Module-3

- 5 a. Find the components of velocity and acceleration of particle whose position vector is $\vec{r} = xi + yi + zk$, where $x = 2 \sin 3t$, $y = 2 \cos 3t$, $z = 8t$ at $t = \frac{\pi}{2}$ along the vector $i + j + k$. (06 Marks)
- b. Find $\text{div } \vec{F}$, $\text{Curl } \vec{F}$ for $\vec{F} = x^2 yzi + y^2 z xj + z^2 xyk$ at $(1, 1, 1)$. (05 Marks)
- c. If ϕ and ψ are any two scalar function. Show that $\text{grad}(\phi \psi) = \phi \text{grad } \psi + \psi \text{grad } \phi$. (05 Marks)

OR

- 6 a. If $\vec{F} = (\sin y + z) \mathbf{i} + (x \cos y - z) \mathbf{j} + (x - y) \mathbf{k}$. Show that \vec{F} is irrotational. Hence find scalar potential ϕ such that $\vec{F} = \nabla \phi$. (06 Marks)
- b. Find Directional Derivative of scalar function $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ along $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. (05 Marks)
- c. If $\vec{f} = r^n \vec{r}$, where $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = |\vec{r}|$, find $\text{div } \vec{f}$. Hence find n for which \vec{f} is Solenoidal. (05 Marks)

Module-4

- 7 a. Find Reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$. (06 Marks)
- b. Find Orthogonal trajectory of $r^n = a^n \sin n\theta$. (05 Marks)
- c. Solve $(y \log x - 2) y \, dx = x \, dy$. (05 Marks)

OR

- 8 a. Evaluate $\int_0^{\pi} x \sin^5 x \cos^6 x \, dx$. (06 Marks)
- b. Solve $y[2x - y + 1] \, dx + x[3x - 4y + j] \, dy = 0$. (05 Marks)
- c. A body in air at 25°C cools from 100°C to 75°C on 1 minute. Find temperature of body at end of 3 minutes. (05 Marks)

Module-5

- 9 a. Solve by Gauss elimination

$$\begin{aligned} x + y + z &= 6 \\ x - y + 2z &= 5 \\ 3x + y + z &= 8. \end{aligned}$$
 (06 Marks)
- b. Find Spectral (Diagonal) and Modal matrix for

$$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$
 (05 Marks)
- c. Show that Linear Transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Hence find Inverse transformation. (05 Marks)

OR

- 10 a. Using Gauss - Seidel, solve

$$\begin{aligned} 10x + y + z &= 12 \\ x + 10y + z &= 12 \\ x + y + 10z &= 12. \end{aligned}$$
 Taking $(0, 0, 0)$ as Initial Vector. (06 Marks)
- b. Using Rayleigh's Power method, find Numerically largest Eigen value and corresponding Eigen vector.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$
 taking $x_0 = (1, 1, 1)'$, carry out 5 iteration. (05 Marks)
- c. Reduce quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ onto Canonical form. (05 Marks)
